निम्न अवकल समीकरणों को हल कीजिए—

1. $\frac{dy}{dx} + \frac{x-2y}{2x-y} = 0$

2. $(x^2 + y^2)\,dx - 2xy\,dy = 0$

3. $(x^2 - y^2)\,dx + 2xy\,dy = 0$

4. $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$

5. $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

6. $\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}$

7. $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$

8. $\frac{dy}{dx} = \frac{2x+9y-20}{6x+2y-10}$

9. $\frac{dy}{dx} = \frac{y-x+1}{y+x+5}$

13. $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$

उत्तरमाला

1. $(x+y)^3 = c\,(x-y)$

2. $x = c\,(x^2 - y^2)$

3. $x^2 + y^2 = cx$

4. $ky = e^{y/x}$

5. $y + \sqrt{x^2+y^2} = cx^2$

6. $\tan\frac{y}{2x} = cx$

7. $\sin\frac{y}{x} = cx$

8. $(2x-y)^2 = c\,(x+2y-5)$

9. $\log\{(y+3)^2 + (x+2)^2\} + 2\tan^{-1}\frac{y+3}{x+2} = c$

13. $\sin\left(\frac{y}{x}\right) = \log cx$

15.2

(1). $\frac{dy}{dx} + \frac{x-2y}{2x-y} = 0$

Put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} + \frac{x-2vx}{2x-vx} = 0$

$\Rightarrow x\frac{dv}{dx} + \frac{1-2v}{2-v} + v = 0$

$\Rightarrow x\frac{dv}{dx} + \frac{1-2v+2v-v^2}{2-v} = 0$

$\Rightarrow x\frac{dv}{dx} + \frac{1-v^2}{2-v} = 0$

$\Rightarrow \frac{(2-v)}{1-v^2}dv + \frac{dx}{x} = 0$

$\Rightarrow \frac{2}{1-v^2}dv - \frac{v\,dv}{1-v^2} + \frac{dx}{x} = 0$

$\int \frac{2}{1-v^2}dv + \int \frac{(-2v)dv}{2(1-v^2} + \int \frac{dx}{x} = 0$

$\Rightarrow \frac{2}{2\times 1}\log\left(\frac{1+v}{1-v}\right) + \frac{1}{2}\log(1-v^2) + \log x = \log c'$

$\Rightarrow \log\left(\frac{1+y/x}{1-y/x}\right) + \frac{1}{2}\log\left(1-\frac{y^2}{x^2}\right) + \log x = \log c'$

$\Rightarrow \log\left(\frac{x+y}{x-y}\right) + \frac{1}{2}\log\left(\frac{x^2-y^2}{x^2}\right) + \log x = \log c'$

$\Rightarrow \log\left\{\left(\frac{x+y}{x-y}\right)\times\frac{\sqrt{x^2-y^2}}{x}\cdot x\right\} = \log c'$

$\Rightarrow \log\left\{\frac{(x+y)^{\frac{3}{2}}}{(x-y)^{\frac{1}{2}}}\right\} = \log c'$

$$\frac{(x+y)^{3/2}}{(x-y)^{\frac{1}{2}}} = C^1$$

$$\frac{(x+y)^3}{(x-y)} = (C^1)^2$$

$$\boxed{(x+y)^3 = C(x-y)} \quad \text{Ans}$$

(9) $(x^2+y^2)\,dx - 2xy\,dy = 0$

$\Rightarrow (x^2+y^2) - 2xy\frac{dy}{dx} = 0$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow (x^2+v^2x^2) - 2x.vx\left(v + x\frac{dv}{dx}\right) = 0$

$\Rightarrow (1+v^2) - 2v^2\left(v + x\frac{dv}{dx}\right) = 0$

$\Rightarrow (1+v^2) - 2v^2 - 2xv\frac{dv}{dx} = 0$

$\Rightarrow (1-v^2) - 2xv\frac{dv}{dx} = 0$

$\Rightarrow \frac{dx}{x} - \frac{2v\,dv}{1-v^2} = 0$

$\Rightarrow \int\frac{dx}{x} + \int\frac{(-2v)\,dv}{(1-v^2)} = \log c$

$\Rightarrow \log x + \log(1-v^2) = \log c$

$\Rightarrow \log x(1-v^2) = \log c$

$\Rightarrow \log x\left(1-\frac{y^2}{x^2}\right) = \log c$

$\Rightarrow \log\frac{(x^2-y^2)}{x} = \log c$

$\Rightarrow \frac{x^2-y^2}{x} = c$ $\Rightarrow x^2-y^2 = cx$ ✓

③ $(x^2-y^2)dx + 2xy\,dy = 0$

$\Rightarrow (x^2-y^2) + 2xy\frac{dy}{dx} = 0$

$(x^2-v^2x^2) + 2x.vx\left(v+x\frac{dv}{dx}\right) = 0$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow (1-v^2) + 2v\left(v+x\frac{dv}{dx}\right) = 0$

$\Rightarrow 1 - v^2 + 2v^2 + 2vx\frac{dv}{dx} = 0$

$\Rightarrow (1+v^2) + 2vx\frac{dv}{dx} = 0$

$\Rightarrow \frac{dx}{x} + \frac{2v\,dv}{1+v^2} = 0$

$\Rightarrow \int\frac{dx}{x} + \int\frac{2v\,dv}{1+v^2} = \log c$

$\Rightarrow \log x + \log(1+v^2) = \log c$

$\Rightarrow \log x(1+v^2) = \log c$

$\Rightarrow \log x\left(1+\frac{y^2}{x^2}\right) = \log c$

$\Rightarrow \log x\left(\frac{x^2+y^2}{x^2}\right) = \log c$

$\Rightarrow \left(\frac{x^2+y^2}{x}\right) = c$

$x^2+y^2 = cx$ Ans

(4) $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$

$\Rightarrow y^2 = (xy - x^2)\frac{dy}{dx}$

$\Rightarrow v^2x^2 = (x^2v - x^2)\left(v + x\frac{dv}{dx}\right)$ | $y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v^2 = (v-1)\left(v + x\frac{dv}{dx}\right)$

$\frac{v^2}{v-1} = v + x\frac{dv}{dx}$

$\Rightarrow \frac{v^2}{v-1} - v = x\frac{dv}{dx}$

$\Rightarrow \frac{v^2 - v^2 + v}{v-1} = x\frac{dv}{dx}$

$\Rightarrow \frac{dx}{x} = \frac{(v-1)dv}{v}$

$\Rightarrow \frac{dx}{x} = \left(1 - \frac{1}{v}\right)dv$

$\Rightarrow \int\frac{dx}{x} + \log c = \int\left(1 - \frac{1}{v}\right)dv$

$\Rightarrow \log x + \log c = v - \log v$

$\Rightarrow \log cx + \log v = v$

$\Rightarrow \log ax + \log\left(\frac{y}{x}\right) = y/x$

$\Rightarrow \log cx \cdot \frac{y}{x} = y/x$

$cy = e^{y/x}$ Ans

(5). $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

$$x\frac{dy}{dx} - y = \sqrt{x^2+y^2}$$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

or $x\left(v + x\frac{dv}{dx}\right) - vx = \sqrt{x^2+v^2x^2}$

or $v + x\frac{dv}{dx} - v = \sqrt{1+v^2}$

or $\int \frac{1}{\sqrt{1+v^2}}\,dv = \int \frac{dx}{x} +$

$$\int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x} + \log c$$

or $\log\left(v + \sqrt{1+v^2}\right) = \log x + \log c$

or $\log\left(\frac{y}{x} + \frac{\sqrt{x^2+y^2}}{x}\right) = \log x + \log c$

or $\log\left(\frac{y+\sqrt{x^2+y^2}}{x}\right) = \log cx$

or $\frac{y+\sqrt{y^2+x^2}}{x} = cx$

$y + \sqrt{y^2+x^2} = cx^2$ Ans

(6) $\frac{dy}{dx} = \frac{y}{x} + \sin(y/x)$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{vx}{x} + \sin v$

or $v + x\frac{dv}{dx} = v + \sin v$

or. $\frac{dv}{\sin v} = \frac{dx}{x}$

$$\frac{d\theta}{\sin\theta} = \frac{dx}{x}$$

$$\int \frac{d\theta}{\sin\theta} = \int \frac{dx}{x} + \log c$$

$$\int \operatorname{cosec}\theta \, d\theta = \int \frac{dx}{x} + \log c$$

$$\log \tan(\theta/2) = \log x + \log c$$

$$\log \tan\left(\frac{y}{2x}\right) = \log cx$$

$$\tan(y/2x) = cx$$

(7) $\frac{dy}{dx} = \frac{y}{x} + \tan(y/x)$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = v + \tan v$$

$$\frac{dv}{\tan v} = \frac{dx}{x}$$

$$\int \cot v \, dv = \int \frac{dx}{x} + \log c$$

$$\log \sin v = \log x + \log c$$

$$\log \sin v = \log cx$$

$$\sin v = cx$$

$$\sin(y/x) = cx$$ Ans

(9) $\frac{dy}{dx} = \frac{y-x+1}{y+x+5}$

but
$x = X+h$
$y = Y+k$

$\frac{dy}{dx} = \frac{Y+k-X-h+1}{Y+k+X+h+5} = \frac{Y-X+k-h+1}{Y+X+k+h+5}$

$\Rightarrow \frac{dy}{dx} = \frac{Y-X}{Y+X}$

$k-h+1=0$
$k+h+5=0$
$-2h = 4$
$h = -2$
$k = -3$

$x = X-2$
$y = Y-3$

but $Y = vX$
$\frac{dY}{dX} = v + X\frac{dv}{dX}$

$v + X\frac{dv}{dX} = \frac{vX-X}{vX+X}$

$v + X\frac{dv}{dX} = \frac{v-1}{v+1}$

$X\frac{dv}{dX} = \frac{v-1}{v+1} - v$

$\Rightarrow X\frac{dv}{dX} = \frac{v-1-v^2-v}{v+1} = -\frac{(1+v^2)}{1+v}$

$\Rightarrow \frac{(1+v)dv}{1+v^2} + \frac{dx}{X} = 0$

$\Rightarrow \int \frac{1}{1+v^2}dv + \int \frac{2v}{(1+v^2)}dv + \int 2\frac{dx}{X} = \log c$ c

~~$2\tan^{-1}v + \frac{1}{2}\log(1+v^2) + \log X$~~

~~$\tan^{-1}(Y/X) + \frac{1}{2}\log\left(\frac{X^2+Y^2}{X^2}\right) + \log X$~~

$\Rightarrow 2\tan^{-1}v + \log(1+v^2) + 2\log X = \log c$

$\Rightarrow 2\tan^{-1}(Y/X) + \log\left(\frac{X^2+Y^2}{X^2}\right) + \log X^2 = \log c$

$\Rightarrow 2\tan^{-1}(Y/X) + \log(X^2+Y^2) = \log c$

$\Rightarrow 2\tan^{-1}\left(\frac{y+3}{x+2}\right) + \log(x^2+4+4x+y^2+9+6y) = \log c$

$\Rightarrow 2\tan^{-1}\left(\frac{y+3}{x+2}\right) + \log(x^2+y^2+4x+6y+13) = C$

$2\tan^{-1}\left(\frac{y+3}{x+2}\right) + \log\{(x+2)^2+(y+3)^2\} = C$ Ans

(13) $x \cos(y/x) \frac{dy}{dx} = y \cos(y/x) + x$

$$x \cos v \left(v + x \frac{dv}{dx}\right) = vx \cos v + x$$

$$\cos v \left(v + x \frac{dv}{dx}\right) = v \cos v + 1$$

$$x \cos v \frac{dv}{dx} = 1$$

$$\int \cos v \, dv = \int \frac{dx}{x} + \log c$$

$$\sin v = \log x + \log c$$

$$\sin(y/x) = \log cx$$ Ans

$y = vx$

$\frac{dy}{dx} = v + x \frac{dv}{dx}$